Collaborative Beamforming for Agents with Localization Errors

Authors: Erfaun Noorani, Yagiz Savas, Alec Koppel, John Baras, Ufuk Topcu, and Brian M. Sadler

Presenter: Yagiz Savas

2021 Asilomar Conference on Signals, Systems, and Computers

Paper ID: 1146

TEXAS
The University of Texas at Austin
UNIVERSITY OF MARYLAND
DEVCOM ARMY RESEARCH LABORATORY

A summary of the paper

Setting

Isotropic antennas

Far-field receiver

Free space

Gaussian localization errors

No feedback from receiver

Goal

Choose a subset of agents such that the expected SNR at the receiver is above a desired threshold Γ with minimum variance.

$$\min_{S \subseteq [N]} \quad Var\left(G(S, \hat{\delta})\right)$$

$$s.t. \quad \mathbb{E}\left[G(S, \hat{\delta})\right] \geq \Gamma$$

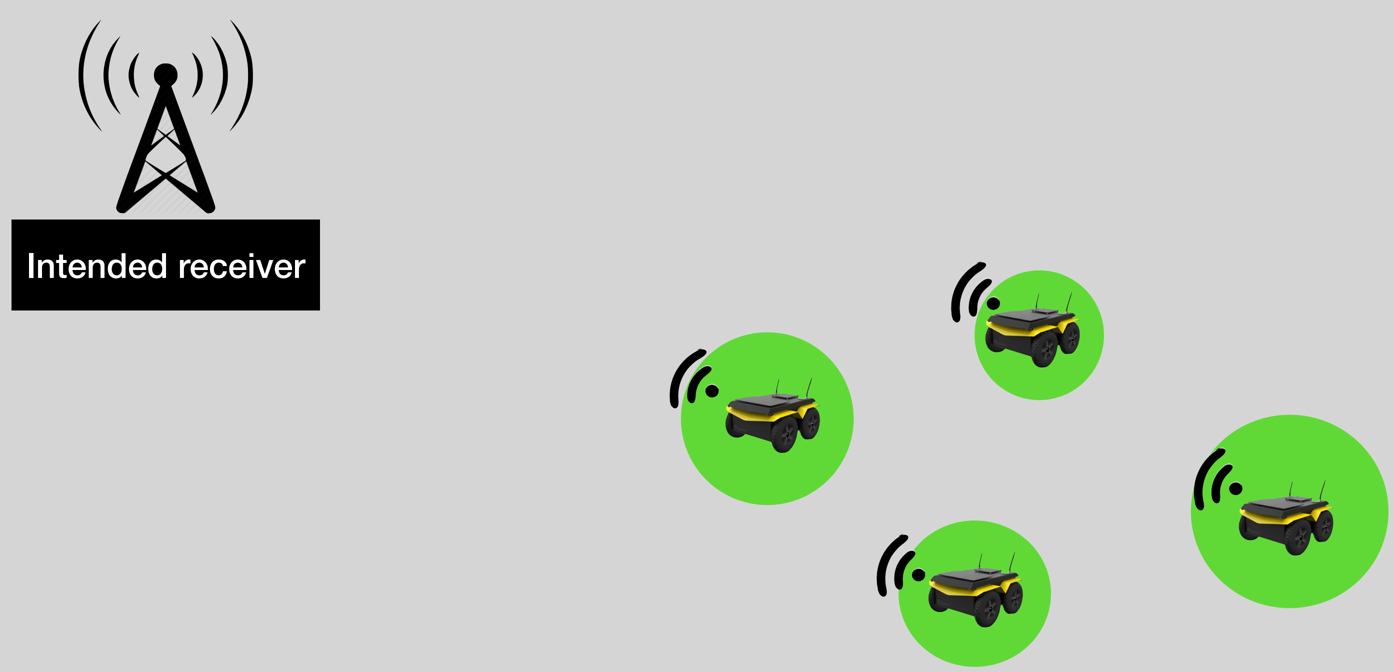
Results

Greedy algorithm: starting with the agent with minimum effective localization variance, keep including them in beamforming until attaining the desired threshold.

Optimality guarantees: variance of the SNR is minimized if all agents have small effective localization variances.

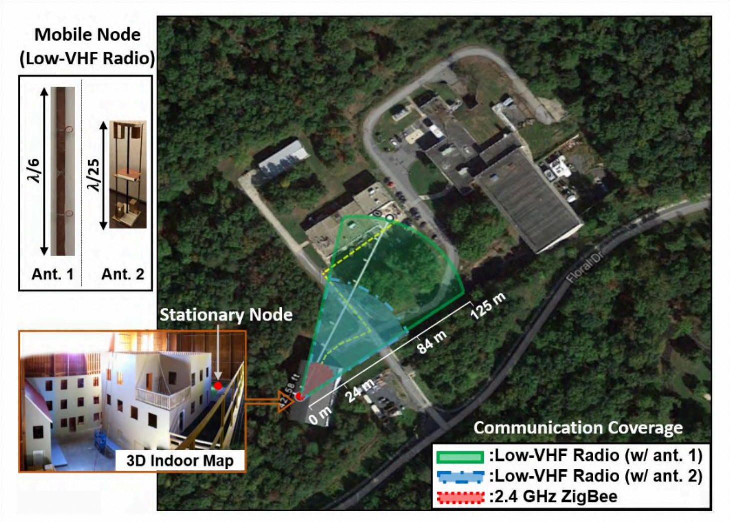
Overview

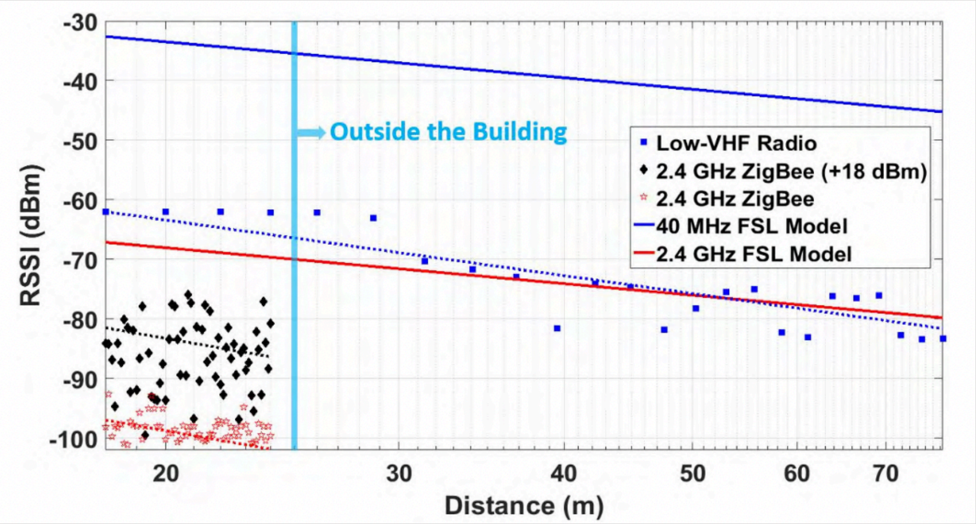
- Assumptions
- Transmission model
- Related work
- A risk-sensitive discrete optimization problem
- A greedy approach for small localization errors
- Extensions for arbitrary localization errors



Assumptions on the communication system

- The base station (BS) is located in the known far-field direction.
- The signal propagates in free space. (~40 MHz)
- The agents' are frequency- and time-synchronized. (short-range radio protocol)
- No mutual coupling between the agents' antennas. (large inter-agent distances)
- The communication takes place over a narrowband wireless channel $h_i = a_i e^{j\eta_i}$.
- All channels attenuate the signal at the same level, i.e., $a_i = a_j$. (similar agent-BS distances)

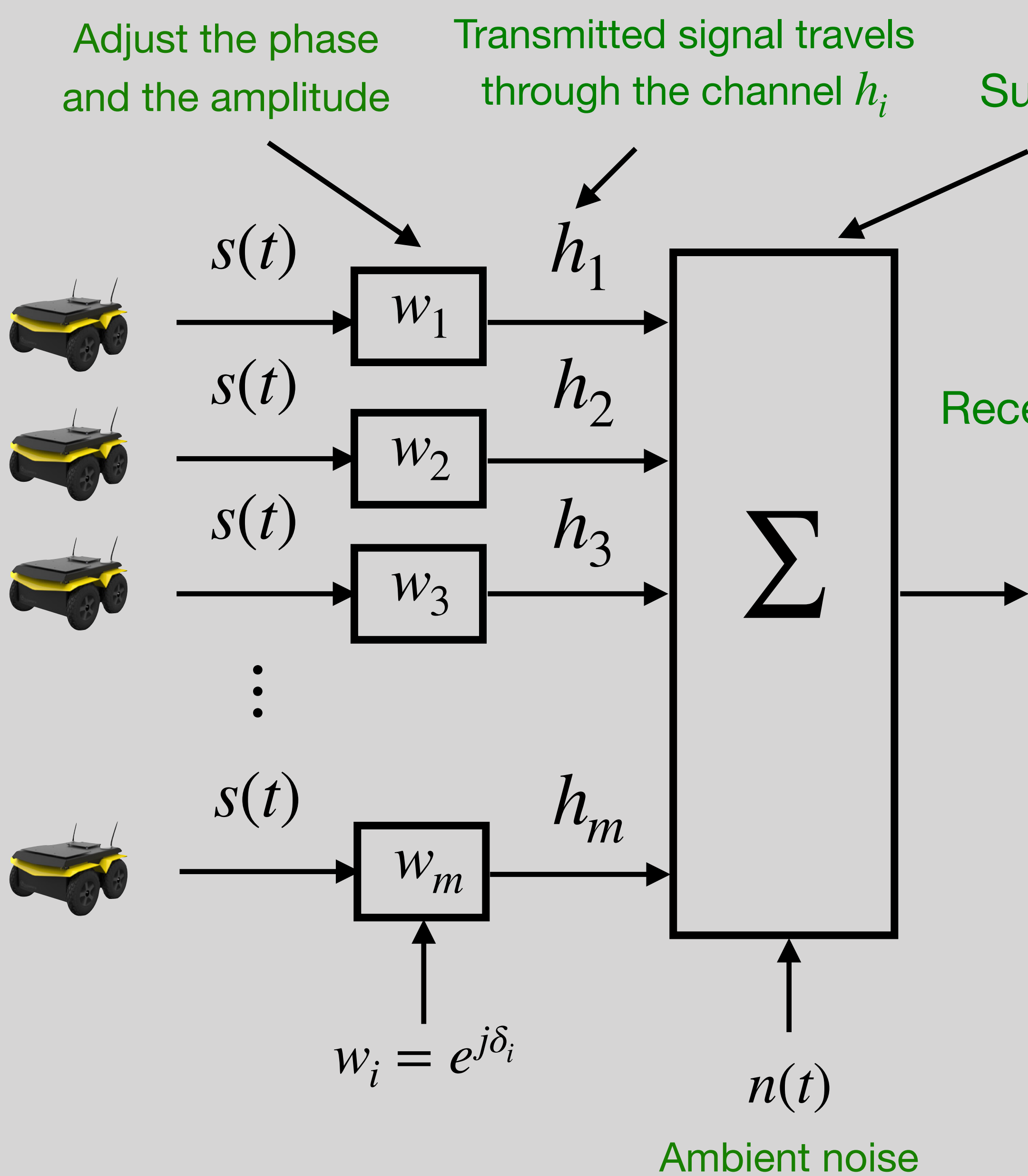




Choi et al., "Low-Power Low-VHF ad-hoc networking in complex environments", IEEE Access, 2017.

Dagefu et al., "Short-range low-VHF channel characterization in cluttered environments", IEEE Transactions on Antennas and Propagation, 2015.

Transmission model



$$y(t) = \sum_{i=1}^{m} w_i h_i s(t) + n(t)$$

Beamforming gain

$$G(S, \delta) = \left| \sum_{i \in S} w_i h_i \right|^2 \propto \left| \sum_{i \in S} e^{j(\delta_i + \eta_i)} \right|^2$$

Related work

- Feedback-based approaches [1]

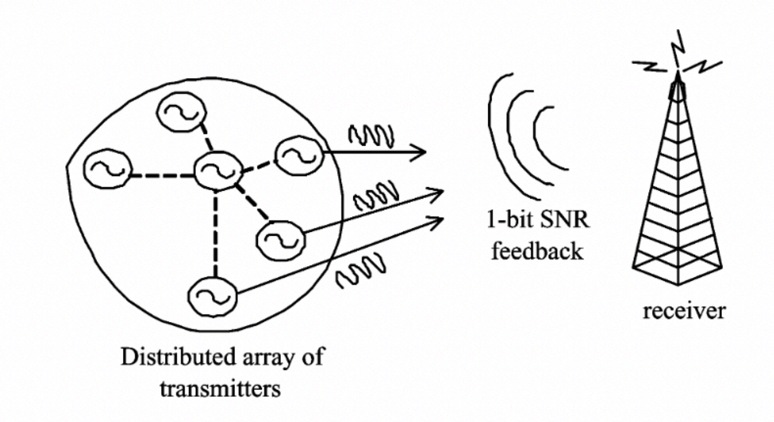


Image from [1]

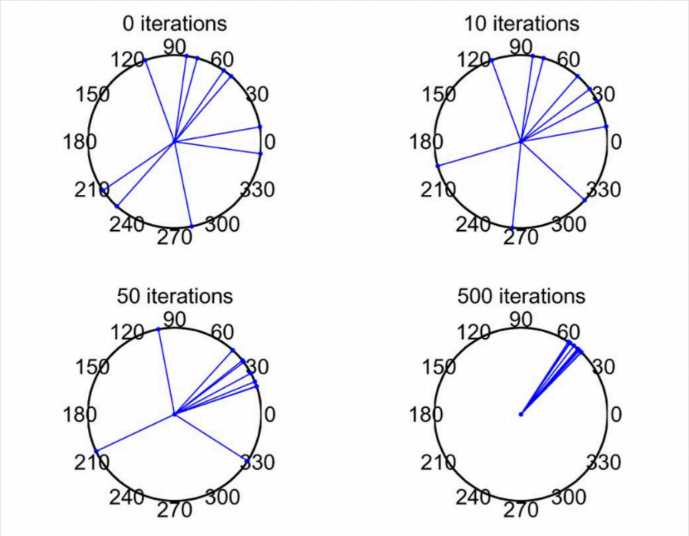


Image from [1]

- Convex optimization-based approaches [2,3,4]

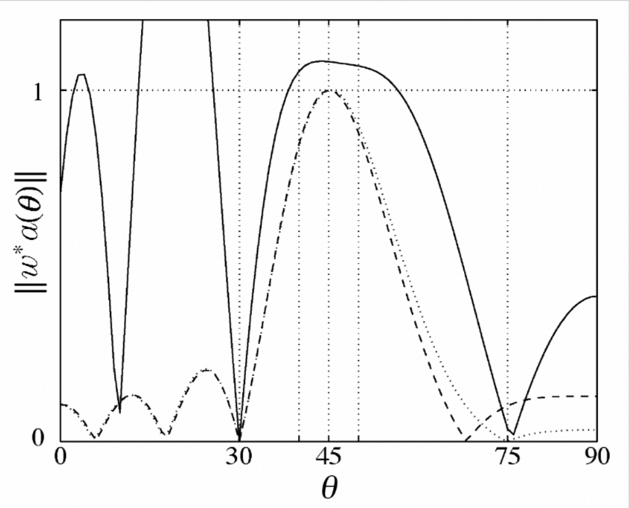


Image from [4]

[1] Mudumbai et al., “Distributed transmit beamforming using feedback control”, IEEE Transactions on Information Theory, 2010.

[2] Wang et al., “Outage Constrained Robust Transmit Optimization for Multiuser MISO Downlinks: Tractable Approximations by Conic Optimization”, IEEE Transactions on Signal Processing, 2014.

[3] Gershman et al., “Convex optimization-based beamforming”, IEEE Signal Processing Magazine, 2010.

[4] Lorenz et al., “Robust minimum variance beamforming”, IEEE Transactions on Signal Processing, 2005.

How to optimize the beamforming gain

$$G(S,\delta) = \left| \sum_{i \in S} e^{j(\delta_i + \eta_i)} \right|^2$$

$$\eta_i = -\frac{2\pi f_c}{C} \langle \vec{r}_i, \vec{r}_{BS} \rangle$$

Local position of the agent $i \in [N]$

- If we knew the local position $\vec{r}_i$ for each $i \in [N]$, then

$$(S^\star, \delta^\star) \in \arg\max_{(S,\delta)} \left| \sum_{i \in S} e^{j(\delta_i + \eta_i)} \right| \iff S^\star = [N] \text{ and } \delta_i^\star = -\eta_i \text{ for each } i \in [N]$$

Include all the agents in beamforming and align their phases

The question that we want to answer:

How to optimize $G(S,\delta)$ if we know the distribution of $\vec{r}_i$?

We will assume that

$$\vec{r}_i \sim N(\mu_i, \Sigma_i)$$

A risk-sensitive optimization problem

- Since the agents' locations $r_i \sim N(\mu_i, \Sigma_i)$, the beamforming gain is a random variable!

- If we just want to maximize the expected beamforming gain, then

$$(\hat{S}, \hat{\delta}) \in \arg\max_{(S,\delta)} \mathbb{E}[G(S, \delta)] \qquad \Longleftrightarrow \qquad \hat{S} = [N] \text{ and } \hat{\delta}_i = -\mathbb{E}[\eta_i] \text{ for each } i \in [N]$$

Include all the agents in beamforming and align their phases in expectation

- We will fix the agent's phases by choosing $\delta_i = \hat{\delta}_i$, and focus on a risk-sensitive formulation

$$\min_{S \subseteq [N]} \quad Var\left(G(S, \hat{\delta})\right)$$
$$s.t. \quad \mathbb{E}\left[G(S, \hat{\delta})\right] \geq \Gamma$$

Intuitively, we want to choose a subset of agents that will form a reliable communication link with high probability

A greedy algorithm and its optimality guarantees

Let $\Phi_i = \hat{\delta}_i + \eta_i$ be the total phase. Then, we have $G(S, \hat{\delta}) = \sum_{i \in S} \sum_{j \in S} cos(\Phi_i - \Phi_j)$.

Recall that $\eta_i = -\frac{2\pi f_c}{C} \langle \vec{r}_i, \vec{r}_{BS} \rangle$ and $\vec{r}_i \sim N(\mu_i, \Sigma_i)$. Then, we have $\Phi_i \sim N(0, \gamma_i)$.

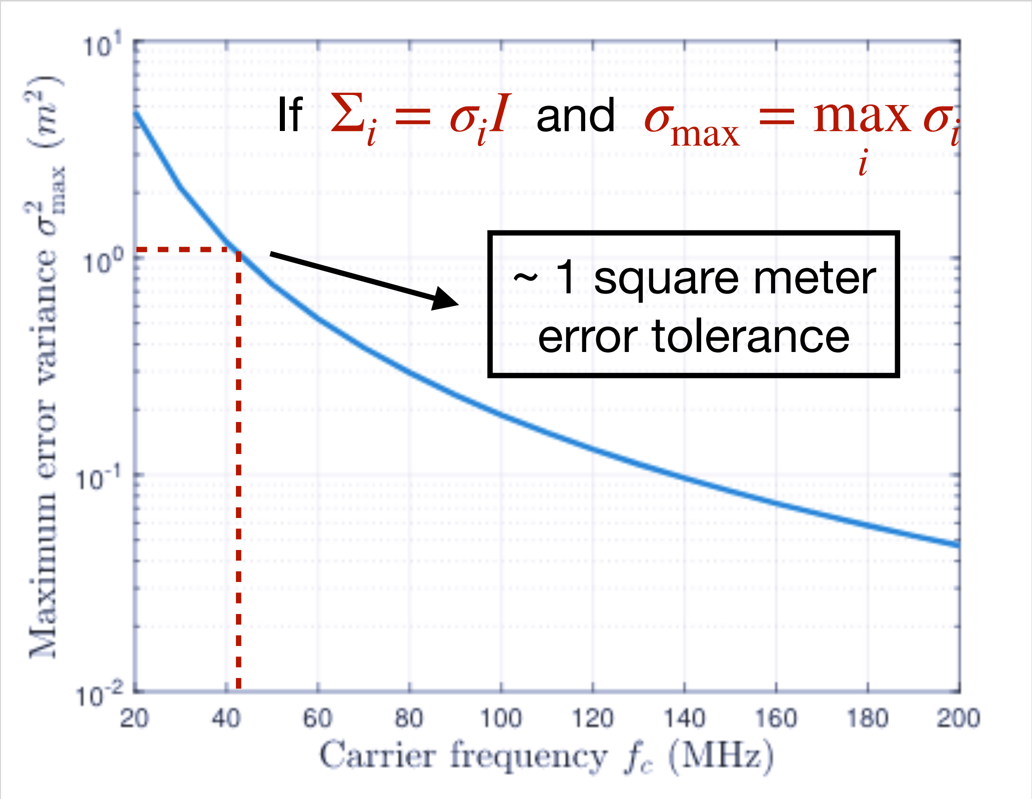
Greedy algorithm:

1. Sort the *effective variances* γ_i in increasing order
2. Add the agents in beamforming until $\mathbb{E}[G(S, \hat{\delta})] \geq \Gamma$

Sufficient conditions for optimality:

1. $\mathbb{E}[G(S, \hat{\delta})] \geq \Gamma$ for $|S| = 2$
2. $\max_i \gamma_i \leq 0.83$

Defines a small error regime!



Extensions to arbitrary localization errors

Recall that we aim to solve:

$$\min_{S \subseteq [N]} \quad Var\Big(G(S, \hat{\delta})\Big)$$

$$s.t. \quad \mathbb{E}\Big[G(S, \hat{\delta})\Big] \geq \Gamma$$

We showed that both $Var\Big(G(S, \hat{\delta})\Big)$ and $\mathbb{E}\Big[G(S, \hat{\delta})\Big]$ are supermodular set functions.

A difference-of-submodular formulation
with local optimality guarantees [1]:

$$\lambda_k = \alpha \lambda_{k-1}, \qquad \alpha > 1, \ \ \lambda_0 > 0$$

$$\min_{S \subseteq [N]} \quad Var\Big(G(S, \hat{\delta})\Big) - \lambda_k \mathbb{E}\Big[G(S, \hat{\delta})\Big]$$

[1] Savas et al., “Collaborative Beamforming Under Localization Errors: A Discrete Optimization Approach”, Signal Processing, (under review)

Comparison with a convex optimization-based beamformer

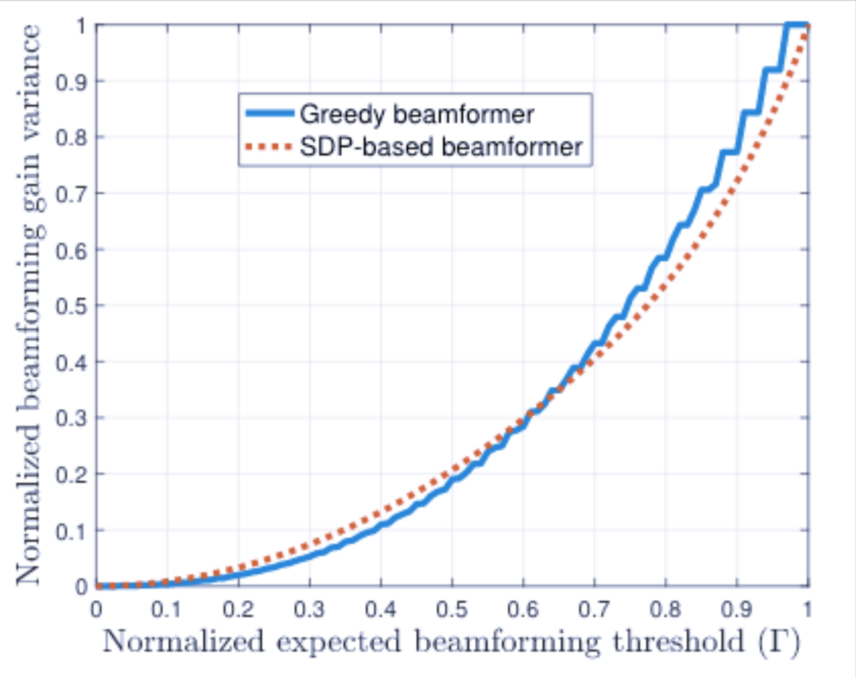
Convex relaxation of
the discrete optimization problem

$$w^\star \in \arg\min_{w \in \mathbb{C}^N} \|w\|^2$$
$$s.t. \quad \mathbb{E}[w^H H w] \geq \Gamma$$
$$\forall i \in [N] \quad |w(i)|^2 \leq 1$$

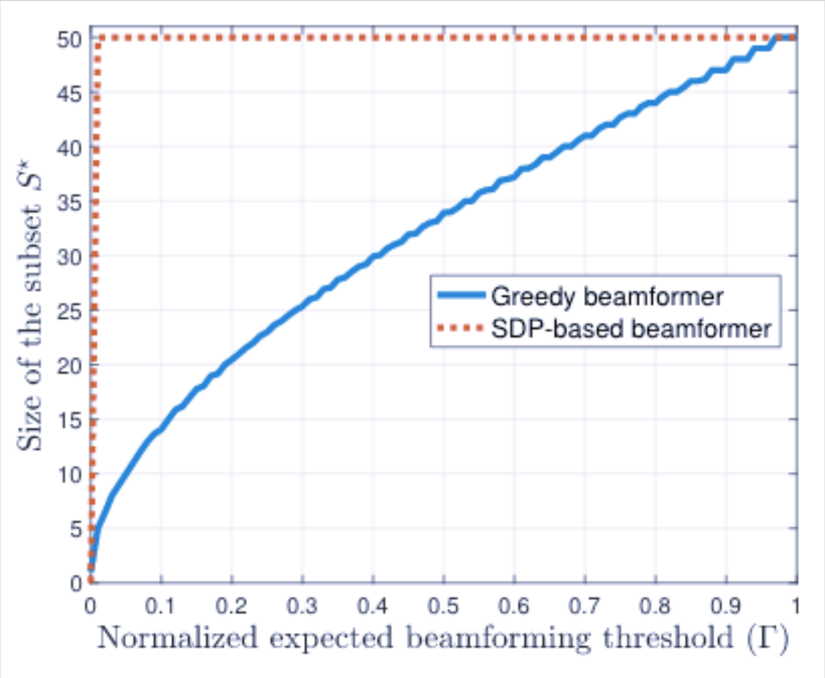
Simulation parameters

$N = 50$ agents
$f_c = 40$ MHz
$\max_i \gamma_i \leq 0.8$
$M = 100$ samples

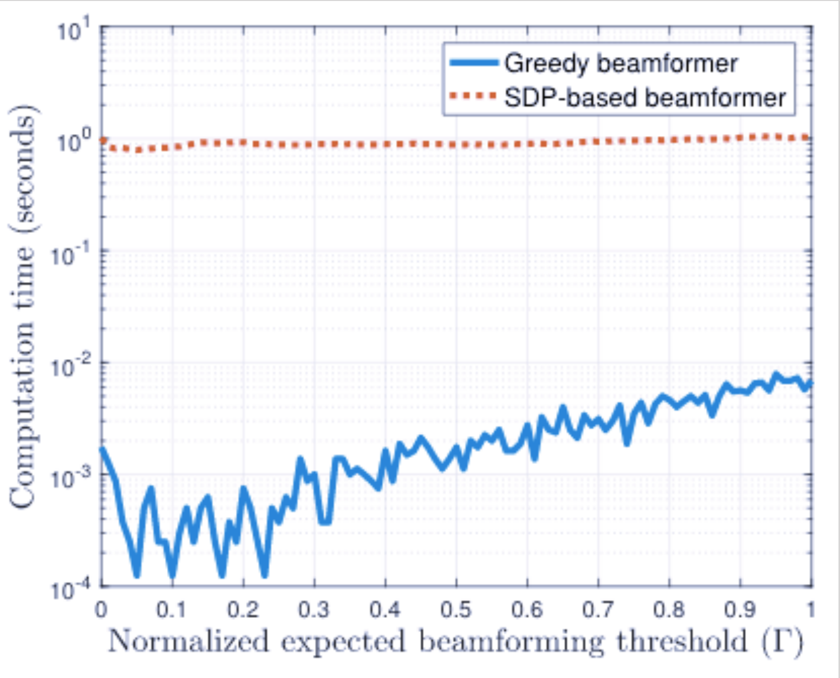
Variance increases superlinearly with respect to the expected gain



Greedy approach achieves the same performance with less agents

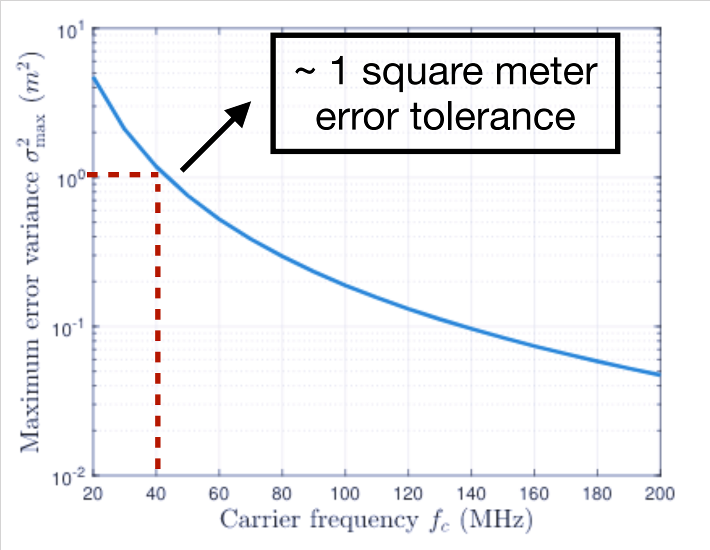


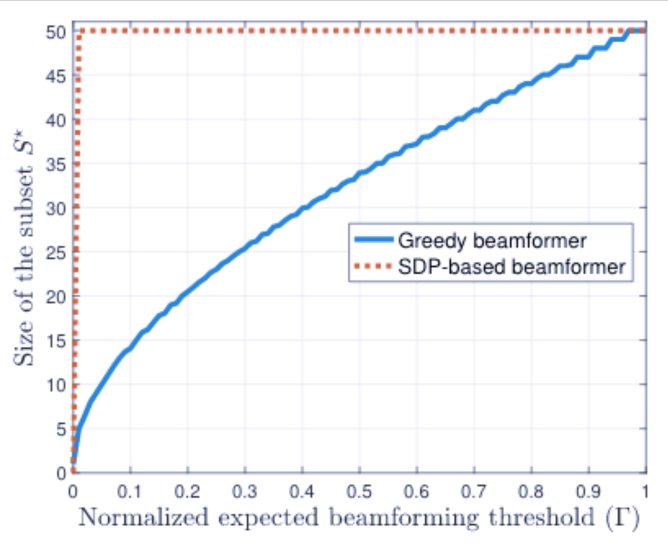
Greedy approach is orders of magnitude faster than the SDP-based approach

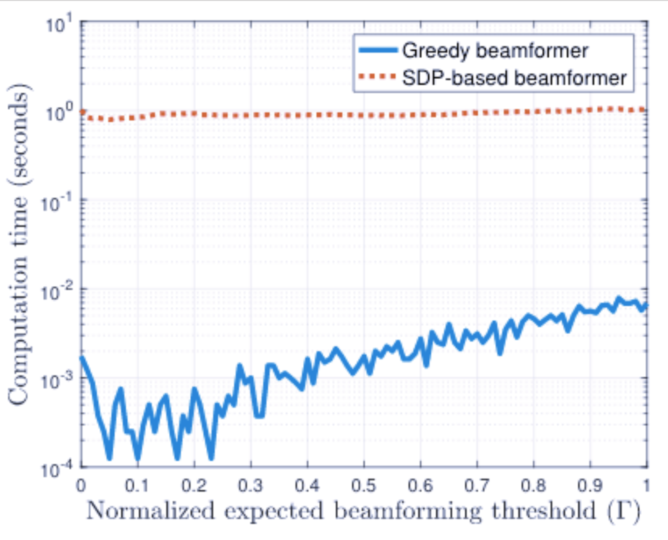


Conclusions

- Collaborative beamforming under Gaussian localization errors
- Risk-sensitive discrete optimization problem
    - To ensure the reliability of communication using minimum number of agents
- A greedy algorithm with global optimality guarantees in the small error regime
    - Orders of magnitude faster than convex optimization-based approaches and utilizes significantly less number of agents to achieve a similar performance







Thank you for listening…

Yagiz Savas

Email: yagiz.savas@utexas.edu